

# $1/m$ corrections to heavy baryon masses in the heavy quark effective theory sum rules

YUAN-BEN DAI<sup>a</sup>, CHAO-SHANG HUANG<sup>a</sup>, CHUN LIU<sup>b,a</sup>, CAI-DIAN LÜ<sup>b,a</sup>

<sup>a</sup> *Institute of Theoretical Physics, Academia Sinica*

*P.O. Box 2735, Beijing 100080, China*<sup>1</sup>

<sup>b</sup> *CCAST (World Laboratory) P.O. Box 8730, Beijing, 100080*

## Abstract

The  $1/m$  corrections to heavy baryon masses are calculated from the QCD sum rules within the framework of the heavy quark effective theory. Numerical results for the heavy baryons are obtained. The implications of the results are discussed.

*PACS:* 12.38.Lg, 12.39.Hg, 14.20.Lq, 14.20.Mr.

*Keywords:*  $1/m_Q$  correction, heavy baryon mass, heavy quark effective theory, QCD sum rule.

---

<sup>1</sup>Mailing address

Heavy baryons provide us a testing ground to the Standard Model (SM), especially to QCD in some aspects. With the accumulation of the experimental data on the heavy baryons, more reliable theoretical calculations are needed, although some of them are rather complicated. Within the framework of the heavy quark effective theory (HQET) which is a model-independent method, the theoretical analysis to the heavy baryons containing a single heavy quark is comparatively simple because of the heavy quark symmetry [1]. However there are still quantities in this framework which need to be determined from nonperturbative QCD.

QCD sum rule [2], which is regarded as a nonperturbative method rooted in QCD itself, has been used successfully to calculate the properties of various hadrons. For instances, besides the light mesons [2], light baryons were first considered in Ref. [3]. Heavy meson properties were systematically analyzed within the HQET [4,5]. Heavy baryons were first discussed in Ref. [6], then masses and Isgur-Wise function for heavy baryons were calculated in the HQET to the leading order heavy quark expansion in Refs. [7] and [8]. In Ref. [9], the calculation for the heavy baryons began with the full theory and results of the calculation were expanded by heavy quark masses. In this paper, within the framework of the HQET, we study the heavy baryonic two-point correlators to the subleading order of the heavy quark expansion by QCD sum rule and obtain results for the heavy baryon masses to that order.

In the HQET, the heavy quark mass  $m_Q$  which is defined perturbatively as the pole mass has been removed by the field redefinition. The heavy quark field  $h_v$  is defined by

$$P_+ Q(x) = \exp(-im_Q v \cdot x) h_v(x) , \quad (1)$$

where  $P_+ = \frac{1}{2}(1 + \not{v})$ . To the order of  $1/m_Q$ , the effective Lagrangian for the heavy quark is [4]

$$\mathcal{L}_{\text{eff}} = \bar{h}_v i v \cdot D h_v + \frac{1}{2m_Q} \bar{h}_v (iD)^2 h_v - \frac{g}{4m_Q} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v . \quad (2)$$

As for the  $1/m_Q$  terms, the first one still conserves heavy quark spin symmetry. It is the last term which violates the spin symmetry. The heavy baryon mass  $M$  is expanded as [10]

$$M = m_Q + \bar{\Lambda} + \frac{\delta\Lambda^K}{m_Q} + \frac{\delta\Lambda^G}{m_Q} \langle \vec{s}_Q \cdot \vec{j}_l \rangle + O(\frac{1}{m_Q^2}) , \quad (3)$$

where  $\bar{\Lambda}$  is the heavy baryon mass in the heavy quark limit, which has been calculated in Ref. [7].  $\delta\Lambda^K$  and  $\delta\Lambda^G$  parameterize the spin-conserved and spin-violated  $1/m_Q$  corrections respectively. All of them characterize the properties of the light degrees of freedom.  $s_Q$  denotes the heavy quark spin, and  $j_l$  stands for the total angular momentum of the light degrees of freedom. For  $\Lambda_Q$  baryon,  $\delta\Lambda^G$  term vanishes. For  $\Sigma_Q^{(*)}$  baryons, both  $\delta\Lambda^K$  and  $\delta\Lambda^G$  terms are nonvanishing with  $\langle \vec{s}_Q \cdot \vec{j}_l \rangle = -1$  for  $\Sigma_Q$  and  $\frac{1}{2}$  for  $\Sigma_Q^*$ .

The heavy baryonic currents  $\tilde{j}^v$  have been given in Refs. [6] and [7] in the rest frame of the heavy baryons. Generally they can be expressed as

$$\tilde{j}^v = \epsilon^{abc} (q_1^{Ta} C \Gamma \tau q_2^b) \Gamma' h_v^c , \quad (4)$$

where  $C$  is the charge conjugate matrix,  $\tau$  is a flavor matrix,  $\Gamma$  and  $\Gamma'$  are some gamma matrices, and  $a, b, c$  denote the color indices.  $\Gamma$  and  $\Gamma'$  can be chosen covariantly as

$$\Gamma_\Lambda = \gamma_5 \quad \Gamma'_\Lambda = 1 , \quad (5)$$

for  $\Lambda_Q$  baryon;

$$\Gamma_\Sigma = \gamma^\mu \quad \Gamma'_\Sigma = (v^\mu + \gamma^\mu) \gamma_5 , \quad (6)$$

for  $\Sigma_Q$  baryon;

$$\Gamma_{\Sigma^*} = \gamma^\nu \quad \Gamma'_{\Sigma^*} = -g^{\mu\nu} + \frac{1}{3}\gamma^\mu \gamma^\nu - \frac{1}{3}(\gamma^\mu v^\nu - \gamma^\nu v^\mu) + \frac{2}{3}v^\mu v^\nu , \quad (7)$$

for  $\Sigma_Q^*$  baryon. The choice of  $\Gamma$  is not unique. Another kind of baryonic current can be obtained by inserting a factor  $\not{v}$  before the  $\Gamma$  in Eqs. (5-7). The currents given by Eqs. (5-7) are denoted as  $\tilde{j}_1^v$ , and that with  $\not{v}$  insertion as  $\tilde{j}_2^v$ . We define the "baryonic

decay constant"  $f$  in the HQET as follows

$$\begin{aligned} \langle 0|\tilde{j}^v|\Lambda_Q \rangle &= f_\Lambda u, \\ \langle 0|\tilde{j}^v|\Sigma_Q \rangle &= f_\Sigma u, \\ \langle 0|\tilde{j}_\mu^v|\Sigma_Q^* \rangle &= \frac{1}{\sqrt{3}}f_{\Sigma^*}u_\mu, \end{aligned} \quad (8)$$

where  $u$  is the spinor and  $u_\mu$  is the Rarita-Schwinger spinor in the HQET respectively.  $f_{\Sigma^*}$  is the same as  $f_\Sigma$  in the heavy quark limit. As the mass expansion (3), the square of  $f$  can be expanded in the same way,

$$f^2 = \bar{f}^2 + \frac{\delta f^{K^2}}{m_Q} + \frac{\delta f^{G^2}}{m_Q} \langle \vec{s}_Q \cdot \vec{j}_l \rangle + O\left(\frac{1}{m_Q^2}\right), \quad (9)$$

where  $\bar{f}^2$  denotes the leading order result and  $\delta f^{K^2}$  and  $\delta f^{G^2}$  the spin-conserved and spin-violated  $1/m_Q$  corrections respectively.

The two-point correlator  $\Gamma(\omega)$  which we choose for sum rule analyzing in the HQET is

$$\Gamma_{ij}(\omega) = i \int d^4x e^{ikx} \langle 0|T\tilde{j}_i^v(x)\bar{\tilde{j}}_j^v(0)|0 \rangle, \quad i, j = 1, 2, \quad (10)$$

where  $\omega = 2v \cdot k$ . The hadronic representation of this correlator is

$$\Gamma_{ij}(\omega) = \left( \frac{2\bar{f}^2}{2\bar{\Lambda} - \omega} - \frac{1}{m_Q} \frac{4\bar{f}^2\delta\bar{\Lambda}}{(2\bar{\Lambda} - \omega)^2} + \frac{1}{m_Q} \frac{2\delta f^2}{2\bar{\Lambda} - \omega} \right)_{ij} \frac{1+\not{v}}{2} + \text{res.}, \quad (11)$$

where  $\delta\bar{\Lambda}$  and  $\delta f^2$  stand for the  $1/m_Q$  corrections in Eqs. (3) and (9). On the other hand,  $\Gamma_{ij}(\omega)$  can be calculated in terms of quark and gluon language with vacuum condensates. This establishes the sum rule. We use the commonly adopted quark-hadron duality for the resonance part of Eq. (11),

$$\text{res.} = \frac{1}{\pi} \int_{\omega_c}^{\infty} d\omega' \frac{\text{Im } \Gamma_{ij}^{\text{pert}}(\omega')}{\omega' - \omega}, \quad (12)$$

where  $\Gamma_{ij}^{\text{pert}}(\omega)$  denotes the perturbative contribution, and  $\omega_c$  is the continuum threshold. In this work, we shall consider only the diagonal correlators ( $i = j$ ).

The calculations of  $\Gamma(\omega)$  are straightforward. The fixed point gauge is used [11]. All the condensates with dimensions lower than 6 are retained. We also include the

dimension 6 condensate  $\langle \bar{q}(0)q(x) \rangle^2$  in our analysis which is a main contribution. We use the gaussian ansatz for the distribution in spacetime for this condensate [12]. In the heavy quark limit, we have double checked the analysis of Ref. [7]. We use the following values of the condensates,

$$\begin{aligned} \langle \bar{q}q \rangle &\simeq -(0.23 \text{ GeV})^3, \\ \langle \alpha_s GG \rangle &\simeq 0.04 \text{ GeV}^4, \\ \langle g\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q \rangle &\equiv m_0^2 \langle \bar{q}q \rangle, \quad m_0^2 \simeq 0.8 \text{ GeV}^2. \end{aligned} \tag{13}$$

When  $\omega_c$  lies between 2.1 – 2.7 GeV for  $\Lambda_Q$  and between 2.3 – 2.9 GeV for  $\Sigma_Q^{(*)}$ , the stability window exists. We obtain

$$\begin{aligned} \bar{\Lambda}_\Lambda &= 0.79 \pm 0.05 \text{ GeV}, \\ \bar{f}_\Lambda^2 &= (0.3 \pm 0.1) \times 10^{-3} \text{ GeV}^6, \end{aligned} \tag{14}$$

for  $\Lambda_Q$  baryon; and

$$\begin{aligned} \bar{\Lambda}_\Sigma &= 0.96 \pm 0.05 \text{ GeV}, \\ \bar{f}_\Sigma^2 &= (1.7 \pm 0.5) \times 10^{-3} \text{ GeV}^6, \end{aligned} \tag{15}$$

for  $\Sigma_Q^{(*)}$  baryon. The normalization  $\text{Tr}\tau^\dagger\tau = 1$  has been used in the analysis. The errors quoted in Eqs. (14) and (15) contain only those from the stability of the sum rule windows. We still do not know the  $\alpha_s$  corrections for baryons. Taking the meson system as a reference [4], the  $\alpha_s$  correction is very small for  $\bar{\Lambda}$ , however it is very large (30%) for  $\bar{f}$ . The numerical results are in agreement with that of Ref. [7], the range of the Borel parameter is the same  $T = 0.4 - 0.7$ .

The  $1/m_Q$  corrections to the two-point correlator  $\Gamma(\omega)$  can be calculated by including insertions of the  $1/m_Q$  operators of the Lagrangian (2) with standard method which is shown in Fig. 1. The insertions of spin-conserved and spin-violated operators are calculated separately. The final form of the sum rules are obtained by performing Borel transformation. With some simple tricks [13], the sum rules for the mass and  $f$  can be separated. The results for the mass of  $\Lambda_Q$  baryon come from the spin-conserved

operators only ( $\delta\Lambda_\Lambda = \delta\Lambda_\Lambda^K$ ):

$$\begin{aligned}\delta\Lambda_{\Lambda_1} &= -\frac{T^2}{16\bar{f}_\Lambda^2} \frac{d}{dT} I_{\Lambda_1} , \\ \delta\Lambda_{\Lambda_2} &= -\frac{T^2}{16\bar{f}_\Lambda^2} \frac{d}{dT} I_{\Lambda_2} ,\end{aligned}\tag{16}$$

where

$$\begin{aligned}I_{\Lambda_1} &= \left( \frac{3}{2^7\pi^4 5} \int_0^{\omega_c} d\omega \omega^6 e^{-\omega/T} + \frac{m_0^2 \langle \bar{q}q \rangle^2}{T} e^{-\frac{m_0^2}{2T^2}} + \frac{43 \langle \alpha_s GG \rangle}{2^5\pi^3 9} T^3 \right) e^{2\bar{\Lambda}/T} , \\ I_{\Lambda_2} &= \left( \frac{1}{2^7\pi^4} \int_0^{\omega_c} d\omega \omega^6 e^{-\omega/T} + \frac{m_0^2 \langle \bar{q}q \rangle^2}{T} e^{-\frac{m_0^2}{2T^2}} + \frac{61 \langle \alpha_s GG \rangle}{2^5\pi^3 9} T^3 \right) e^{2\bar{\Lambda}/T} ,\end{aligned}\tag{17}$$

and the subscripts 1 and 2 denote  $\tilde{j}_1^v$  and  $\tilde{j}_2^v$  respectively. The sum rule for  $f$  of  $\Lambda_Q$  is

$$\delta f_{\Lambda_i}^2 = -\frac{1}{8} \left( 1 + \frac{d}{d \ln T} \right) I_{\Lambda_i} .\tag{18}$$

The masses of baryons  $\Sigma_Q$  and  $\Sigma_Q^*$  are given in terms of  $\delta\Lambda^K$  and  $\delta\Lambda^G$ . They are determined by the following sum rules,

$$\begin{aligned}\delta\Lambda_{\Sigma_1}^K &= -\frac{T^2}{16\bar{f}_\Sigma^2} \frac{d}{dT} I_{\Sigma_1}^K , \\ \delta\Lambda_{\Sigma_2}^K &= -\frac{T^2}{16\bar{f}_\Sigma^2} \frac{d}{dT} I_{\Sigma_2}^K , \\ \delta\Lambda_{\Sigma_1}^G = \delta\Lambda_{\Sigma_2}^G &= -\frac{T^2}{16\bar{f}_\Sigma^2} \frac{d}{dT} I_\Sigma^G ,\end{aligned}\tag{19}$$

where

$$\begin{aligned}I_{\Sigma_1}^K &= \left( \frac{11}{2^7\pi^4 5} \int_0^{\omega_c} d\omega \omega^6 e^{-\omega/T} + \frac{3m_0^2 \langle \bar{q}q \rangle^2}{T} e^{-\frac{m_0^2}{2T^2}} + \frac{13 \langle \alpha_s GG \rangle}{2^5\pi^3 3} T^3 \right) e^{2\bar{\Lambda}/T} , \\ I_{\Sigma_2}^K &= \left( \frac{13}{2^7\pi^4 5} \int_0^{\omega_c} d\omega \omega^6 e^{-\omega/T} + \frac{3m_0^2 \langle \bar{q}q \rangle^2}{T} e^{-\frac{m_0^2}{2T^2}} - \frac{5 \langle \alpha_s GG \rangle}{2^5\pi^3 3} T^3 \right) e^{2\bar{\Lambda}/T} , \\ I_\Sigma^G &= \frac{\langle \alpha_s GG \rangle}{4\pi^3} T^3 e^{2\bar{\Lambda}/T} .\end{aligned}\tag{20}$$

The sum rules for  $f$  are given by

$$\delta f_{\Sigma_i}^{2K,G} = -\frac{1}{8} \left( 1 + \frac{d}{d \ln T} \right) I_{\Sigma(i)}^{K,G} .\tag{21}$$

It can be seen that while the two diagonal sum rules coincided with each other at the leading order, they are no longer the same for the spin-conserved  $1/m_Q$  corrections.

The numerical sum rule results for the  $1/m_Q$  corrections –  $\delta\Lambda$  and  $\delta f^2$  in Eqs. (3) and (9) are given in Tables 1 and 2 and Figs. 2-4. The numerical differences resulting from the different choices of  $\tilde{j}^v$  are not significant. The values of  $\omega_c$  are generally smaller than the leading order results, but still lie in the allowed range of the leading order results. The lower limit of the Borel parameter  $T = 0.4$  is determined by requiring that the condensates in Eqs. (17) and (20) have less than 40% contribution. The upper limit  $T = 0.6$  is obtained by requiring that the pole contribution is over 70%. This window is narrower than the leading order one. In the window  $T = 0.4 - 0.6$  the results for  $\delta\Lambda_\Lambda$  and  $\delta\Lambda_\Sigma^K$  are comparatively stable. However from Fig. 4, we see that  $\delta\Lambda_\Sigma^G$  has no good stability in this window. This is because we have not included the Feynman diagrams with internal gluon lines which are expected to be important for the spin-violated terms. Therefore the value  $\delta\Lambda_\Sigma^G$  in Table 1 is not reliable. The errors quoted in Tables 1 and 2 again only refer to that from the stability of the sum rule windows.

From  $m_{\Lambda_c}$  and  $m_{\Lambda_b}$  [14], we determine the heavy quark masses  $m_c = 1.43 \pm 0.05$  GeV and  $m_b = 4.83 \pm 0.07$  GeV. These values give the following results,

$$m_{\Sigma_c} = 2.52 \pm 0.08 \text{ GeV} , m_{\Sigma_c^*} = 2.55 \pm 0.08 \text{ GeV} , \quad (22)$$

$$m_{\Sigma_b} = 5.83 \pm 0.09 \text{ GeV} , m_{\Sigma_b^*} = 5.84 \pm 0.09 \text{ GeV} . \quad (23)$$

From the discussion above we know the individual mass value in Eqs. (22) and (23) suffers from the inaccuracy of  $\delta\Lambda_\Sigma^G$ . The quantity

$$\frac{1}{3}(m_{\Sigma_Q} + 2m_{\Sigma_Q^*}) = m_Q + \bar{\Lambda} + \frac{1}{m_Q}(0.22 \pm 0.06 \text{ GeV}^2) \quad (24)$$

is independent of  $\delta\Lambda_\Sigma^G$ , therefore more reliable. It is  $2.54 \pm 0.08$  GeV for  $c$  quark case and  $5.83 \pm 0.09$  GeV for  $b$  quark case. Experimentally  $m_{\Sigma_c} = 2453 \pm 0.2$  MeV [14].

There is an experimental evidence for  $\Sigma_c^*$  at  $m_{\Sigma_c^*} = 2530 \pm 7$  MeV [14]. If we take this value for  $m_{\Sigma_c^*}$ , we have  $\frac{1}{3}(m_{\Sigma_c} + 2m_{\Sigma_c^*}) = 2504 \pm 5$  MeV. This is in reasonable agreement with the theoretical value. The corresponding quantity for the bottom quark can be checked by the experiments in the near future.

To conclude, we have calculated the  $1/m_Q$  corrections to the heavy baryon masses from the QCD sum rules within the framework of the HQET. This study refines the leading order analysis [7]. Furthermore within this framework, we can study the three-point correlators which will give the form factors for the weak transitions of the heavy baryons [8] to the order of  $1/m_Q$ . It is also viable to include the QCD radiative corrections in the leading order and subleading order calculations. Both of these two aspects are under our studying.

One of us (Liu) would like to thank M. Chabab, K.T. Chao, W.F. Chen, Y. Liao, M. Tong and especially C.W. Luo for helpful discussions. This work is supported in part by the China Postdoctoral Science Foundation.



## References

- [1] N. Isgur and M.B. Wise, Nucl. Phys. B 348 (1991) 276;  
H. Georgi, Nucl. Phys. B 348 (1991) 293;  
T. Mannel, W. Roberts and Z. Ryzak, Nucl. Phys. B 355 (1991) 38.
- [2] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B 147 (1979) 385;  
B147 (1979) 488.
- [3] B.L. Ioffe, Nucl. Phys. B 188 (1981) 317, Errata B 191 (1981) 591;  
Y. Chung, H.G. Dosch, M. Kremer and D. Schall, Nucl. Phys. B 197 (1982) 55.
- [4] For a review, see M. Neubert, Phys. Rep. 245 (1994) 259.
- [5] P. Ball, Nucl. Phys. B 421 (1994) 593, Phys. Rev. D49 (1994) 2472.
- [6] E.V. Shuryak, Nucl. Phys. B 198 (1982) 83.
- [7] A.G. Grozin and O.I. Yakovlev, Phys. Lett. B 285 (1992) 254.
- [8] A.G. Grozin and O.I. Yakovlev, Phys. Lett. B 291 (1992) 441.
- [9] B. Bagan, M. Chabab, H.G. Dosch and S. Narison, Phys. Lett. B 301 (1993) 243.
- [10] H.Y. Jin, C.S. Huang and Y.B. Dai, Z. Phys. C56 (1992) 707;  
C.S. Huang, to appear in Comm. Theor. Phys. 24 (1995) .
- [11] For a review, see V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov,  
Fortschr. Phys. 32 (1984) 585.
- [12] S.V. Mikhailov and A.V. Radyushkin, Sov. J. Nucl. Phys. 49 (1989) 494.
- [13] M. Neubert, Phys. Rev. D46 (1992) 1076.
- [14] Particle Data Group, Phys. Rev. D50 (1994) 1173.

## Figure captions

Fig. 1. The subleading operator insertions relevant to our analysis.

Fig. 2. Sum rules for  $\delta\Lambda_\Lambda$  with (a)  $\tilde{j}_1^v$  and (b)  $\tilde{j}_2^v$ .  $\omega_c = 2.0, 2.1, 2.4$  GeV for solid, dashed, dash-dotted curves respectively. The sum rule window is  $T = 0.4 - 0.6$  GeV.

Fig. 3. Sum rules for  $\delta\Lambda_\Sigma^K$  with (a)  $\tilde{j}_1^v$  and (b)  $\tilde{j}_2^v$ .  $\omega_c = 2.2, 2.4, 2.7$  GeV for solid, dashed, dash-dotted curves respectively. The sum rule window is  $T = 0.4 - 0.6$  GeV.

Fig. 4. Sum rule for  $\delta\Lambda_\Sigma^G$ . The sum rule window is  $T = 0.4 - 0.6$  GeV.

## Tables

Table 1. Numerical results for  $\delta\Lambda$ .

	$\delta\Lambda_\Lambda(\text{GeV}^2)$	$\delta\Lambda_\Sigma^K(\text{GeV}^2)$	$\delta\Lambda_\Sigma^G(\text{GeV}^2)$
	$\omega_c=2.1 \pm 0.1 \text{ GeV}$	$\omega_c=2.4 \pm 0.2 \text{ GeV}$	
$\tilde{j}_1^v$	$0.09 \pm 0.03$	$0.22 \pm 0.06$	$0.03 \pm 0.02$
$\tilde{j}_2^v$	$0.09 \pm 0.05$	$0.21 \pm 0.06$	$0.03 \pm 0.02$

Table 2. Numerical results for  $\delta f^2$ .

	$\delta f_\Lambda^2(10^{-3}\text{GeV}^6)$	$\delta f_\Sigma^{K^2}(10^{-3}\text{GeV}^6)$	$\delta f_\Sigma^{G^2}(10^{-3}\text{GeV}^6)$
	$\omega_c=2.1 \pm 0.1 \text{ GeV}$	$\omega_c=2.4 \pm 0.2 \text{ GeV}$	
$\tilde{j}_1^v$	$-0.20 \pm 0.1$	$0.7 \pm 0.4$	$-0.1 \pm 0.1$
$\tilde{j}_2^v$	$-0.3 \pm 0.1$	$0.8 \pm 0.5$	$-0.1 \pm 0.1$

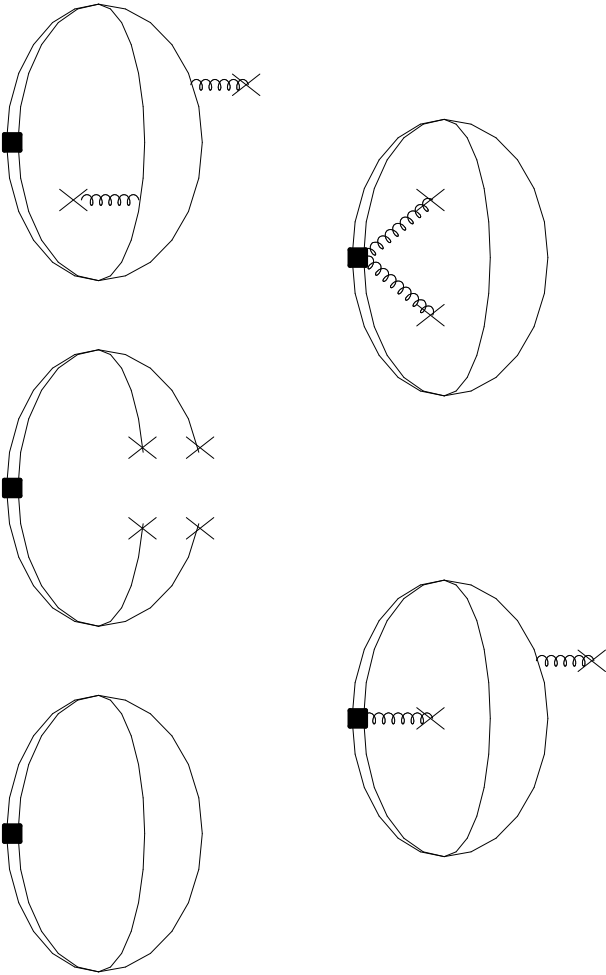


Fig. 1

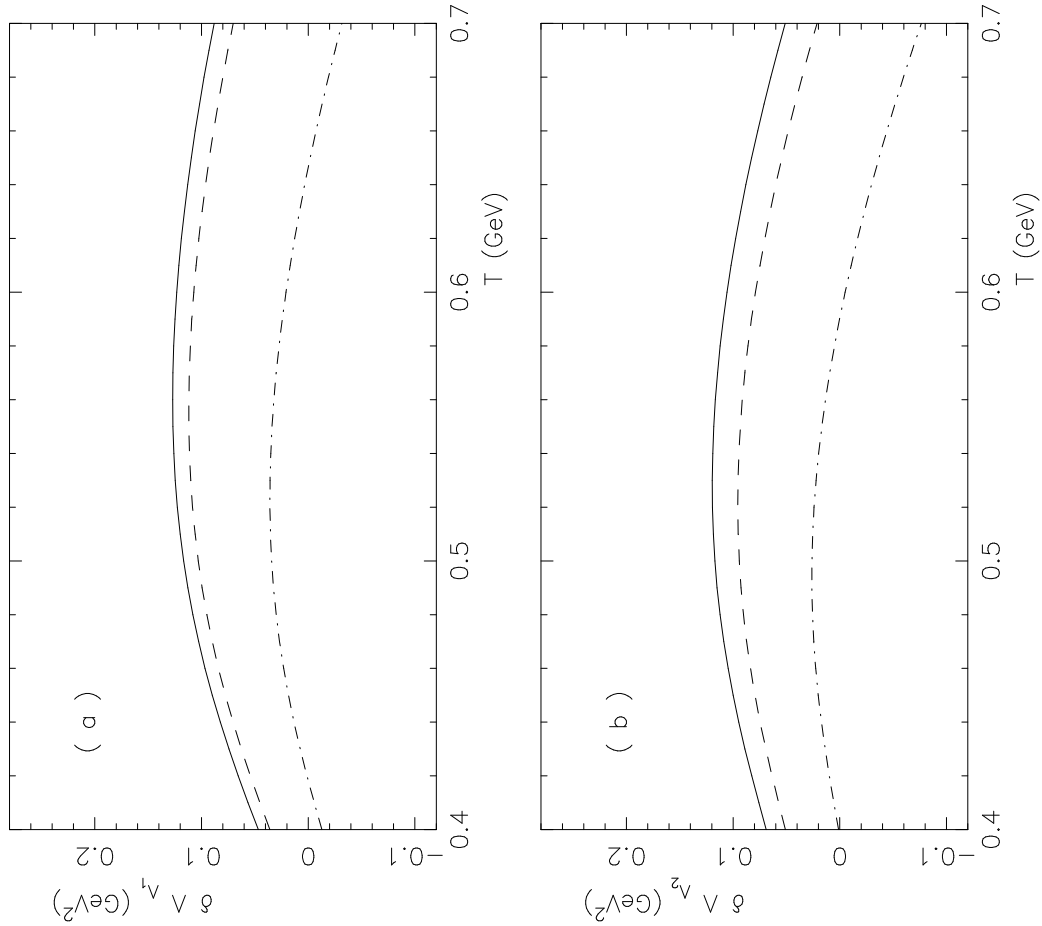


Fig. 2

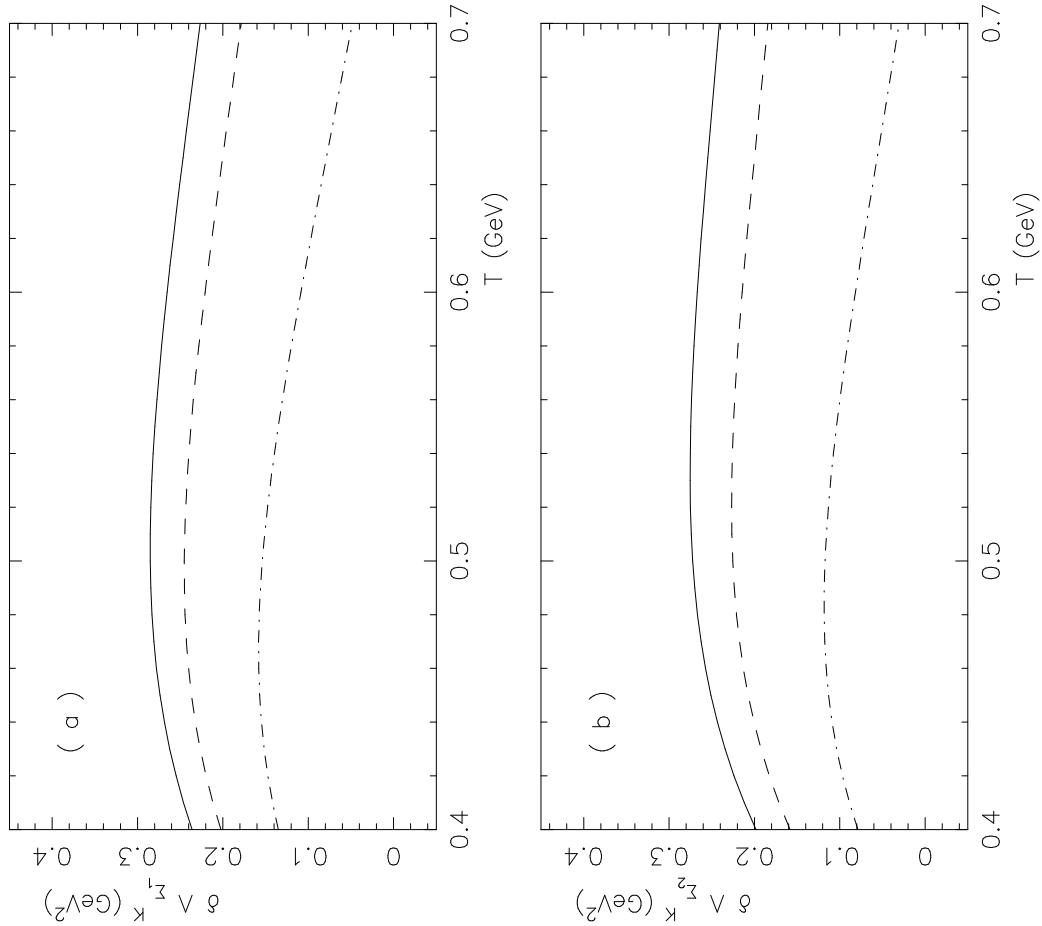


Fig. 3

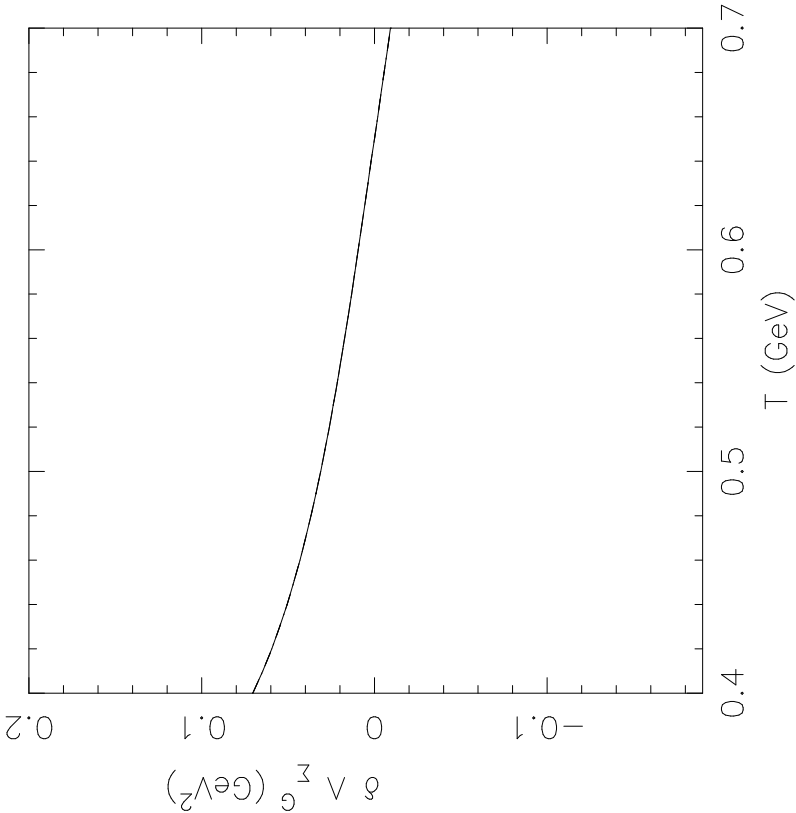


Fig. 4